

THE SEMANTIC CONCEPTION OF TRUTH AND THE FOUNDATIONS OF SEMANTICS

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This paper consists of two parts; the first has an expository character, and the second is rather polemical.

In the first part I want to summarize in an informal way the main results of my investigations concerning the definition of truth and the more general problem of the foundations of semantics. These results have been embodied in a work which appeared in print several years ago.¹ Although my investigations concern concepts dealt with in classical philosophy, they happen to be comparatively little known in philosophical circles, perhaps because of their strictly technical character. For this reason I hope I shall be excused for taking up the matter once again.²

Since my work was published, various objections, of unequal value, have been raised to my investigations; some of these appeared in print, and others were made in public and private discussions in which I took part.³ In the second part of the paper I should like to express my views regarding these objections. I hope that the remarks which will be made in this context will not be considered as purely polemical in character, but will be found to contain some constructive contributions to the subject.

In the second part of the paper I have made extensive use of material graciously put at my disposal by Dr. Marja Kokoszynska (University of Lwow). I am especially indebted and grateful to Professors Ernest Nagel (Columbia University) and David Rynin (University of California, Berkeley) for their help in preparing the final text and for various critical remarks.

I. EXPOSITION

1. THE MAIN PROBLEM -- A SATISFACTORY DEFINITION OF TRUTH.

Our discussion will be centered around the notion⁴ of *truth*. The main problem is that of giving a *satisfactory definition* of this notion, i.e., a definition which is *materially adequate* and *formally correct*. But such a formulation of the problem, because of its generality, cannot be considered unequivocal, and requires some further comments.

In order to avoid any ambiguity, we must first specify the conditions under which the definition of truth will be considered adequate from the material point of view. The desired definition does not aim to specify the meaning of a familiar word used to denote a novel notion; on the contrary, it aims to catch hold of the actual meaning of an old notion. We must then characterize this notion precisely enough to enable anyone to determine whether the definition actually fulfills its task.

Secondly, we must determine on what the formal correctness of the definition depends. Thus, we must specify the words or concepts which we wish to use in defining the notion of truth; and we must also give the formal rules to which the definition should conform. Speaking more generally, we must describe the formal structure of the language in which the definition will be given.

The discussion of these points will occupy a considerable portion of the first part of the paper.

2. THE EXTENSION OF THE TERM "TRUE."

We begin with some remarks regarding the extension of the concept of truth which we have in mind here.

The predicate "*true*" is sometimes used to refer to psychological phenomena such as judgments or beliefs, sometimes to certain physical objects, namely, linguistic expressions and specifically sentences, and sometimes to certain ideal entities called "propositions." By "sentence" we understand here what is usually meant in grammar by "declarative sentence"; as regards the term "proposition," its meaning is notoriously a subject of lengthy disputations by various philosophers and logicians, and it seems never to have been made quite clear and unambiguous. For several reasons it appears most convenient to *apply the term "true" to sentences*, and we shall follow this course.⁵

Consequently, we must always relate the notion of truth, like that of a sentence, to a specific language; for it is obvious that the same expression which is a true sentence in one language can be false or meaningless in another.

Of course, the fact that we are interested here primarily in the notion of truth for sentences does not exclude the possibility of a subsequent extension of this notion to other kinds of objects.

3. THE MEANING OF THE TERM "TRUE."

Much more serious difficulties are connected with the problem of the meaning (or the intension) of the concept of truth.

The word "*true*," like other words from our everyday language, is certainly not unambiguous. And it does not seem to me that the philosophers who have discussed this concept have helped to diminish its ambiguity. In works and discussions of philosophers we meet many different conceptions of truth and falsity, and we must indicate which conception will be the basis of our discussion.

We should like our definition to do justice to the intuitions which adhere to the *classical Aristotelian conception of truth* -- intuitions which find their expression in the well-known words of Aristotle's *Metaphysics*:

To say of what is that it is not, or of what is not that it is, is false, while to say of what is that it is, or of what is not that it is not, is true.

If we wished to adapt ourselves to modern philosophical terminology, we could perhaps express this conception by means of the familiar formula

The truth of a sentence consists in its agreement with (or correspondence to) reality.
(For a theory of truth which is to be based upon the latter formulation the term "correspondence theory" has been suggested.)

If, on the other hand, we should decide to extend the popular usage of the term "*designate*" by applying it not only to names, but also to sentences, and if we agreed to speak of the designata of sentences as "states of affairs," we could possibly use for the same purpose the following phrase:

*A sentence is true if it designates an existing state of affairs.*⁶

However, all these formulations can lead to various misunderstandings, for none of them is sufficiently precise and clear (though this applies much less to the original Aristotelian formulation than to either of the others); at any rate, none of them can be considered a satisfactory definition of truth. It is up to us to look for a more precise expression of our intuitions.

4. A CRITERION FOR THE MATERIAL ADEQUACY OF THE DEFINITION.⁷

Let us start with a concrete example. Consider the sentence "*snow is white*." We ask the question under what conditions this sentence is true or false. It seems clear that if we base ourselves on the classical conception of truth, we shall say that the sentence is true if snow is white, and that it is false if snow is not white. Thus, if the definition of truth is to conform to our conception, it must imply the following equivalence:

The sentence "snow is white" is true if, and only if, snow is white.

Let me point out that the phrase "*snow is white*" occurs on the left side of this equivalence in quotation marks, and on the right without quotation marks. On the right side we have the sentence itself, and on the left the name of the sentence. Employing the medieval logical terminology we could also say that on the right side the words "*snow is white*" occur in *suppositio formalis*, and on the left in *suppositio materialis*. It is hardly necessary to explain why we must have the name of the sentence, and not the sentence itself, on the left side of the equivalence. For, in the first place, from the point of view of the grammar of our language, an expression of the form "*X is true*" will not become a meaningful sentence if we replace in it 'X' by a sentence or by anything other than a name -- since the subject of a sentence may be only a noun or an expression functioning like a noun. And, in the second place, the fundamental conventions regarding the use of any language require that in any utterance we make about an object it is the name of the object which must be employed, and not the object itself. In consequence, if we wish to say something about a sentence, for example, that it is true, we must use the name of this sentence, and not the sentence itself.⁸

It may be added that enclosing a sentence in quotation marks is by no means the only way of forming its name. For instance, by assuming the usual order of letters in our alphabet, we can use the following expression as the name (the description) of the sentence "*snow is white*":

the sentence constituted by three words, the first of which consists of the 19th, 14th, 15th, and 23rd letters, the second of the 9th and 19th letters, and the third of the 23rd, 8th, 9th, 20th, and 5th letters of the English alphabet.

We shall now generalize the procedure which we have applied above. Let us consider an arbitrary sentence; we shall replace it by the letter '*p*.' We form the name of this sentence and we replace it by another letter, say '*X*.' We ask now what is the logical relation between the two sentences "*X is true*" and '*p*.' It is clear that from the point of view of our basic conception of truth these sentences are equivalent. In other words, the following equivalence holds:

(T) *X is true if, and only if, p.*

We shall call any such equivalence (with '*p*' replaced by any sentence of the language to which the word "*true*" refers, and '*X*' replaced by a name of this sentence) an "*equivalence of the form (T).*"

Now at last we are able to put into a precise form the conditions under which we will consider the usage and the definition of the term "*true*" as adequate from the material point of view: we wish to use the term "*true*" in such a way that all equivalences of the form (T) can be asserted, and *we shall call a definition of truth "adequate" if all these equivalences follow from it.*

It should be emphasized that neither the expression (T) itself (which is not a sentence, but only a schema of a sentence) nor any particular instance of the form (T) can be regarded as a definition of truth. We can only say that every equivalence of the form (T) obtained by replacing '*p*' by a particular sentence, and '*X*' by a name of this sentence, may be considered a partial definition of truth, which explains wherein the truth of this one individual sentence consists. The general definition has to be, in a certain sense, a logical conjunction of all these partial definitions.

(The last remark calls for some comments. A language may admit the construction of infinitely many sentences; and thus the number of partial definitions of truth referring to sentences of such a language will also be infinite. Hence to give our remark a precise sense we should have to explain what is meant by a "logical conjunction of infinitely many sentences"; but this would lead us too far into technical problems of modern logic.)

5. TRUTH AS A SEMANTIC CONCEPT.

I should like to propose the name "*the semantic conception of truth*" for the conception of truth which has just been discussed.

Semantics is a discipline which, speaking loosely, *deals with certain relations between expressions of a language and the objects (or "states of affairs") "referred to" by those expressions.* As typical examples of semantic concepts we may mention the concepts of *designation*, *satisfaction*, and *definition* as these occur in the following examples:

the expression "the father of his country" designates (denotes) George Washington;

snow satisfies the sentential function (the condition) "2 is white";

the equation " $2 \cdot x = 1$ " defines (uniquely determines) the number $1/2$.

While the words "*designates*," "*satisfies*," and "*defines*" express relations (between certain expressions and the objects "referred to" by these expressions), the word "*true*" is of a different logical nature: it expresses a property (or denotes a class) of certain expressions, viz, of sentences. However, it is easily seen that all the formulations which were given earlier and which aimed to explain the meaning of this word (cf. [Sections 3 and 4](#)) referred not only to sentences themselves, but also to objects "talked about" by these sentences, or possibly to "states of affairs" described by them. And, moreover, it turns out that the simplest and the most natural way of obtaining an exact definition of truth is one which involves the use of other semantic notions, e.g., the notion of satisfaction. It is for these reasons that we count the concept of truth which is discussed here among the concepts of semantics, and the problem of defining truth proves to be closely related to the more general problem of setting up the foundations of theoretical semantics.

It is perhaps worth while saying that semantics as it is conceived in this paper (and in former papers of the author) is a sober and modest discipline which has no pretensions of being a universal patent-medicine for all the ills and diseases of mankind, whether imaginary or real. You will not find in semantics any remedy for decayed teeth or illusions of grandeur or class conflicts. Nor is semantics a device for establishing that everyone except the speaker and his friends is speaking nonsense.

From antiquity to the present day the concepts of semantics have played an important role in the discussions of philosophers, logicians, and philologists. Nevertheless, these concepts have been treated for a long time with a certain amount of suspicion. From a historical standpoint, this suspicion is to be regarded as completely justified. For although the meaning of semantic concepts as they are used in everyday language seems to be rather clear and understandable, still all attempts to characterize this meaning in a general and exact way miscarried. And what is worse, various arguments in which these concepts were involved, and which seemed otherwise quite correct and based upon apparently obvious premises, led frequently to paradoxes and antinomies. It is sufficient to mention here the *antinomy of the liar*, Richard's *antinomy of definability* (by means of a finite number of words), and Grelling-Nelson's *antinomy of heterological terms*.⁹

I believe that the method which is outlined in this paper helps to overcome these difficulties and assures the possibility of a consistent use of semantic concepts.

6. LANGUAGES WITH A SPECIFIED STRUCTURE.

Because of the possible occurrence of antinomies, the problem of specifying the formal structure and the vocabulary of a language in which definitions of semantic concepts are to be given becomes especially acute; and we turn now to this problem.

There are certain general conditions under which the structure of a language is regarded as *exactly specified*. Thus, to specify the structure of a language, we must characterize unambiguously the class of those words and expressions which are to be considered *meaningful*. In particular, we must indicate all words which we decide to use without defining them, and which are called "*undefined (or primitive) terms*"; and we must give the so-called *rules of definition* for introducing new or *defined terms*. Furthermore, we must set up criteria for distinguishing within the class of expressions those which we call "*sentences*." Finally, we must formulate the conditions under which a sentence of the language can be *asserted*. In particular, we must indicate all *axioms (or primitive sentences)*, i.e., those sentences which we decide to assert without proof; and we must give the so-called *rules of inference (or rules of proof)* by means of which we can deduce new asserted sentences from other sentences which have been previously asserted. Axioms, as well as sentences deduced from them by means of rules of inference, are referred to as "*theorems*" or "*provable sentences*."

If in specifying the structure of a language we refer exclusively to the form of the expressions involved, the language is said to be *formalized*. In such a language theorems are the only sentences which can be asserted.

At the present time the only languages with a specified structure are the formalized languages of various systems of deductive logic, possibly enriched by the introduction of certain non-logical terms. However, the field of application of these languages is rather comprehensive; we are able, theoretically, to develop in them various branches of science, for instance, mathematics and theoretical physics.

(On the other hand, we can imagine the construction of languages which have an exactly specified structure without being formalized. In such a language the assertability of sentences, for instance, may depend not always on their form, but sometimes on other, non-linguistic factors. It would be interesting and important actually to construct a language of this type, and specifically one which would prove to be sufficient for the development of a comprehensive branch of empirical science; for this would justify the hope that languages with specified structure could finally replace everyday language in scientific discourse.)

The problem of the definition of truth obtains a precise meaning and can be solved in a rigorous way only for those languages whose structure has been exactly specified. For other languages -- thus, for all natural, "spoken" languages -- the meaning of the problem is more or less vague, and its solution can have only an approximate character. Roughly speaking, the approximation consists in replacing a natural language (or a portion of it in which we are interested) by one whose structure is exactly specified, and which diverges from the given language "as little as possible."

7. THE ANTINOMY OF THE LIAR.

In order to discover some of the more specific conditions which must be satisfied by languages in which (or for which) the definition of truth is to be given, it will be advisable to begin with a discussion of that antinomy which directly involves the notion of truth, namely, the antinomy of the liar.

To obtain this antinomy in a perspicuous form,¹⁰ consider the following sentence:

The sentence printed in this paper on p. 347, l. 31, is not true.

For brevity we shall replace the sentence just stated by the letter 's.'

According to our convention concerning the adequate usage of the term "true," we assert the following equivalence of the form (T):

(1) *'s' is true if, and only if, the sentence printed in this paper on p. 347, l. 31, is not true.*

On the other hand, keeping in mind the meaning of the symbol 's,' we establish empirically the following fact:

(2) *'s' is identical with the sentence printed in this paper on p. 347, l. 31.*

Now, by a familiar law from the theory of identity (Leibniz's law), it follows from (2) that we may replace in (1) the expression "*the sentence printed in this paper on p. 347, l. 31*" by the symbol "'s.'" We thus obtain what follows:

(3) *'s' is true if, and only if, 's' is not true.*

In this way we have arrived at an obvious contradiction.

In my judgment, it would be quite wrong and dangerous from the standpoint of scientific progress to depreciate the importance of this and other antinomies, and to treat them as jokes or sophistries. It is a fact that we are here in the presence of an absurdity, that we have been compelled to assert a false sentence (since (3), as an equivalence between two contradictory sentences, is necessarily false). If we take our work seriously, we cannot be reconciled with this fact. We must discover its cause, that is to say, we must analyze premises upon which the antinomy is based; we must then reject at least one of these premises, and we must investigate the consequences which this has for the whole domain of our research.

It should be emphasized that antinomies have played a preeminent role in establishing the foundations of modern deductive sciences. And just as class-theoretical antinomies, and in particular Russell's antinomy (of the class of all classes that are not members of themselves) were the starting point for the successful attempts at a consistent formalization of logic and mathematics, so the antinomy of the liar and other semantic antinomies give rise to the construction of theoretical semantics.

8. THE INCONSISTENCY OF SEMANTICALLY CLOSED LANGUAGES.⁷

If we now analyze the assumptions which lead to the antinomy of the liar, we notice the following:

- I. We have implicitly assumed that the language in which the antinomy is constructed contains, in addition to its expressions, also the names of these expressions, as well as semantic terms such as the term "*true*" referring to sentences of this language; we have also assumed that all sentences which determine the adequate usage of this term can be asserted in the language. A language with these properties will be called "*semantically closed*."
- II. We have assumed that in this language the ordinary laws of logic hold.
- III. We have assumed that we can formulate and assert in our language an empirical premise such as the statement (2) which has occurred in our argument.

It turns out that the assumption (III) is not essential, for it is possible to reconstruct the antinomy of the liar without its help.¹¹ But the assumptions (I) and (II) prove essential. Since every language which satisfies both of these assumptions is inconsistent, we must reject at least one of them;

It would be superfluous to stress here the consequences of rejecting the assumption (II), that is, of changing our logic (supposing this were possible) even in its more elementary and fundamental parts. We thus consider only the possibility of rejecting the assumption (I). Accordingly, we decide *not to use any language which is semantically closed* in the sense given.

This restriction would of course be unacceptable for those who, for reasons which are not clear to me, believe that there is only one "genuine" language (or, at least, that all "genuine" languages are mutually translatable). However, this restriction does not affect the needs or interests of science in any essential way. The languages (either the formalized languages or -- what is more frequently the case -- the portions of everyday language) which are used in scientific discourse do not have to be semantically closed. This is obvious in case linguistic phenomena and, in particular, semantic notions do not enter in any way into the subject-matter of a science; for in such a case the language of this science does not have to be provided with any semantic terms at all. However, we shall see in the next section how semantically closed languages can be dispensed with even in those scientific discussions in which semantic notions are essentially involved.

The problem arises as to the position of everyday language with regard to this point. At first blush it would seem that this language satisfies both assumptions (I) and (II), and that therefore it must be inconsistent. But actually the case is not so simple. Our everyday language is certainly not one with an exactly specified structure. We do not know precisely, which expressions are sentences, and we know even to a smaller degree which sentences are to be taken as assertible. Thus the problem of consistency has no exact meaning with respect to this language. We may at best only risk the guess that a language whose structure has been exactly specified and which resembles our everyday language as closely as possible would be inconsistent.

9. OBJECT-LANGUAGE AND META-LANGUAGE.

Since we have agreed not to employ semantically closed languages, we have to use two different languages in discussing the problem of the definition of truth and, more generally, any problems in the field of semantics. The first of these languages is the language which is "talked about" and which is the subject matter of the whole discussion; the definition of truth which we are seeking applies to the sentences of this language. The second is the language in which we "talk about" the first language, and in terms of which we wish, in particular, to construct the definition of truth for the first language. We shall refer to the first language as "*the object language*," and to the second as "*the meta-language*."

It should be noticed that these terms "object-language" and "meta-language" have only a relative sense. If, for instance, we become interested in the notion of truth applying to sentences, not of our original object-language, but of its meta-language, the latter becomes automatically the object-language of our discussion; and in order to define truth for this language, we have to go to a new meta-language so to speak, to a meta-language of a higher level. In this way we arrive at a whole hierarchy of languages.

The vocabulary of the meta-language is to a large extent determined by previously stated conditions under which a definition of truth will be considered materially adequate. This definition, as we recall, has to imply all equivalences of the form (T):

(T) *X is true if, and only if, p.*

The definition itself and all the equivalences implied by it are to be formulated in the meta-language. On the other hand, the symbol '*p*' in (T) stands for an arbitrary sentence of our object-language. Hence it follows that every sentence which occurs in the object-language must also occur in the meta-language; in other words, the meta-language must contain the object-language as a part. This is at any rate necessary for the proof of the adequacy of the definition -- even though the definition itself can sometimes be formulated in a less comprehensive meta-language which does not satisfy this requirement.

(The requirement in question can be somewhat modified, for it suffices to assume that the object-language can be translated into the meta-language; this necessitates a certain change in the interpretation of the symbol '*p*' in (T). In all that follows we shall ignore the possibility of this modification.)

Furthermore, the symbol '*X*' in (T) represents the name of the sentence which '*p*' stands for. We see therefore that the meta-language must be rich enough to provide possibilities of constructing a name for every sentence of the object-language.

In addition, the meta-language must obviously contain terms of a general logical character, such as the expression "if, and only if."¹²

It is desirable for the meta-language not to contain any undefined terms except such as are involved explicitly or implicitly in the remarks above, i.e.: terms of the object-language; terms referring to the form of the expressions of the object-language, and

used in building names for these expressions; and terms of logic. In particular, we desire *semantic terms* (referring to the object-language) *to be introduced into the meta-language only by definition*. For, if this postulate is satisfied, the definition of truth, or of any other semantic concept, will fulfill what we intuitively expect from every definition; that is, it will explain the meaning of the term being defined in terms whose meaning appears to be completely clear and unequivocal. And, moreover, we have then a kind of guarantee that the use of semantic concepts will not involve us in any contradictions.

We have no further requirements as to the formal structure of the object-language and the meta-language; we assume that it is similar to that of other formalized languages known at the present time. In particular, we assume that the usual formal rules of definition are observed in the meta-language.

10. CONDITIONS FOR A POSITIVE SOLUTION OF THE MAIN PROBLEM.

Now, we have already a clear idea both of the conditions of material adequacy to which the definition of truth is subjected, and of the formal structure of the language in which this definition is to be constructed. Under these circumstances the problem of the definition of truth acquires the character of a definite problem of a purely deductive nature.

The solution of the problem, however, is by no means obvious, and I would not attempt to give it in detail without using the whole machinery of contemporary logic. Here I shall confine myself to a rough outline of the solution and to the discussion of certain points of a more general interest which are involved in it.

The solution turns out to be sometimes positive, sometimes negative. This depends upon some formal relations between the object-language and its meta-language; or, more specifically, upon the fact whether the meta-language in its logical part is "*essentially richer*" than the object-language or not. It is not easy to give a general and precise definition of this notion of "essential richness." If we restrict ourselves to languages based on the logical theory of types, the condition for the meta-language to be "essentially richer" than the object-language is that it contain variables of a higher logical type than those of the object-language.

If the condition of "essential richness" is not satisfied, it can usually be shown that an interpretation of the meta-language in the object-language is possible; that is to say, with any given term of the meta-language a well determined term of the object-language can be correlated in such a way that the assertible sentences of the one language turn out to be correlated with assertible sentences of the other. As a result of this interpretation, the hypothesis that a satisfactory definition of truth has been formulated in the meta-language turns out to imply the possibility of reconstructing in that language the antinomy of the liar; and this in turn forces us to reject the hypothesis in question.

(The fact that the meta-language, in its non-logical part, is ordinarily more comprehensive than the object-language does not affect the possibility of interpreting the former in the latter. For example, the names of expressions of the object-language

occur in the meta-language, though for the most part they do not occur in the object-language itself; but, nevertheless, it may be possible to interpret these names in term of the object-language.)

Thus we see that the condition of "essential richness" is necessary for the possibility of a satisfactory definition of truth in the meta-language. If we want to develop the theory of truth in a meta-language which does not satisfy this condition, we must give up the idea of defining truth with the exclusive help of those terms which were indicated above (in [Section 8](#)). We have then to include the term "*true*," or some other semantic term, in the list of undefined terms of the meta-language, and to express fundamental properties of the notion of truth in a series of axioms. There is nothing essentially wrong in such an axiomatic procedure, and it may prove useful for various purposes.¹³

It turns out, however, that this procedure can be avoided. For *the condition of the "essential richness" of the meta-language proves to be, not only necessary, but also sufficient for the construction of a satisfactory definition of truth*; i.e., if the meta-language satisfies this condition, the notion of truth can be defined in it. We shall now indicate in general terms how this construction can be carried through.

11. THE CONSTRUCTION (IN OUTLINE) OF THE DEFINITION.¹⁴

A definition of truth can be obtained in a very simple way from that of another semantic notion, namely, of the notion of *satisfaction*.

Satisfaction is a relation between arbitrary objects and certain expressions called "*sentential functions*." These are expressions like "*x is white*," "*x is greater than y*," etc. Their formal structure is analogous to that of sentences; however, they may contain the so-called free variables (like '*x*' and '*y*' in "*x is greater than y*"), which cannot occur in sentences.

In defining the notion of a sentential function in formalized languages, we usually apply what is called a "recursive procedure"; i.e., we first describe sentential functions of the simplest structure (which ordinarily presents no difficulty), and then we indicate the operations by means of which compound functions can be constructed from simpler ones. Such an operation may consist, for instance, in forming the logical disjunction or conjunction of two given functions, i.e., by combining them by the word "*or*" or "*and*." A sentence can now be defined simply as a sentential function which contains no free variables

As regards the notion of satisfaction, we might try to define it by saying that given objects satisfy a given function if the latter becomes a true sentence when we replace in it free variables by names of given objects. In this sense, for example, snow satisfies the sentential function "*x is white*" since the sentence "*snow is white*" is true. However, apart from other difficulties, this method is not available to us, for we want to use the notion of satisfaction in defining truth.

To obtain a definition of satisfaction we have rather to apply again a recursive procedure. We indicate which objects satisfy the simplest sentential functions; and then

we state the conditions under which given objects satisfy a compound function -- assuming that we know which objects satisfy the simpler functions from which the compound one has been constructed. Thus, for instance, we say that given numbers satisfy the logical disjunction "*x is greater than y or x is equal to y*" if they satisfy at least one of the functions "*x is greater than y*" or "*x is equal to y*."

Once the general definition of satisfaction is obtained, we notice that it applies automatically also to those special sentential functions which contain no free variables, i.e., to sentences. It turns out that for a sentence only two cases are possible: a sentence is either satisfied by all objects, or by no objects. Hence we arrive at a definition of truth and falsehood simply by saying that *a sentence is true if it is satisfied by all objects, and false otherwise.*¹⁵

(It may seem strange that we have chosen a roundabout way of defining the truth of a sentence, instead of trying to apply, for instance, a direct recursive procedure. The reason is that compound sentences are constructed from simpler sentential functions, but not always from simpler sentences; hence no general recursive method is known which applies specifically to sentences.)

From this rough outline it is not clear where and how the assumption of the "essential richness" of the meta-language is involved in the discussion; this becomes clear only when the construction is carried through in a detailed and formal way.¹⁶

12. CONSEQUENCES OF THE DEFINITION.

The definition of truth which was outlined above has many interesting consequences.

In the first place, the definition proves to be not only formally correct, but also materially adequate (in the sense established in [Section 4](#)); in other words, it implies all equivalences of the form (T). In this connection it is important to notice that the conditions for the material adequacy of the definition determine uniquely the extension of the term "*true*."¹⁷ Therefore, every definition of truth which is materially adequate would necessarily be equivalent to that actually constructed. The semantic conception of truth gives us, so to speak, no possibility of choice between various non-equivalent definitions of this notion.

Moreover, we can deduce from our definition various laws of a general nature. In particular, we can prove with its help the *laws of contradiction and of excluded middle*, which are so characteristic of the Aristotelian conception of truth; i.e., we can show that one and only one of any two contradictory sentences is true. These semantic laws should not be identified with the related logical laws of contradiction and excluded middle; the latter belong to the sentential calculus, i.e., to the most elementary part of logic and do not involve the term "*true*" at all.

Further important results can be obtained by applying the theory of truth to formalized languages of a certain very comprehensive class of mathematical disciplines; only disciplines of an elementary character and a very elementary logical structure are excluded from this class. It turns out that for a discipline of this class *the notion of truth*

never coincides with that of provability; for all provable sentences are true, but there are true sentences which are not provable.¹⁷ Hence it follows further that every such discipline is consistent but incomplete; that is to say, of two contradictory sentences at most one is provable, and -- what is more -- there exists a pair of contradictory sentences neither of which is provable.¹⁸

13. EXTENSION OF THE RESULTS TO OTHER SEMANTIC NOTIONS.

Most of the results at which we arrived in the preceding sections in discussing the notion of truth can be extended with appropriate changes to other semantic notions, for instance, to the notion of satisfaction (involved in our previous discussion), and to those of *designation* and *definition*.

Each of these notions can be analyzed along the lines followed in the analysis of truth. Thus, criteria for an adequate usage of these notions can be established; it can be shown that each of these notions, when used in a semantically closed language according to those criteria, leads necessarily to a contradiction;¹⁹ a distinction between the object-language and the meta-language becomes again indispensable; and the "essential richness" of the meta-language proves in each case to be a necessary and sufficient condition for a satisfactory definition of the notion involved. Hence the results obtained in discussing one particular semantic notion apply to the general problem of the foundations of theoretical semantics.

Within theoretical semantics we can define and study some further notions, whose intuitive content is more involved and whose semantic origin is less obvious; we have in mind, for instance, the important notions of *consequence*, *synonymity*, and *meaning*.²⁰

We have concerned ourselves here with the theory of semantic notions related to an individual object-language (although no specific properties of this language have been involved in our arguments). However, we could also consider the problem of developing *general semantics* which applies to a comprehensive class of object-languages. A considerable part of our previous remarks can be extended to this general problem; however, certain new difficulties arise in this connection, which will not be discussed here. I shall merely observe that the axiomatic method (mentioned in [Section 10](#)) may prove the most appropriate for the treatment of the problem.²¹

This paper consists of two parts; the first has an expository character, and the second is rather polemical.

In the first part I want to summarize in an informal way the main results of my investigations concerning the definition of truth and the more general problem of the foundations of semantics. These results have been embodied in a work which appeared in print several years ago.¹ Although my investigations concern concepts dealt with in classical philosophy, they happen to be comparatively little known in philosophical circles, perhaps because of their strictly technical character. For this reason I hope I shall be excused for taking up the matter once again.²

Since my work was published, various objections, of unequal value, have been raised to my investigations; some of these appeared in print, and others were made in public and private discussions in which I took part.³ In the second part of the paper I should like to express my views regarding these objections. I hope that the remarks which will be made in this context will not be considered as purely polemical in character, but will be found to contain some constructive contributions to the subject.

In the second part of the paper I have made extensive use of material graciously put at my disposal by Dr. Marja Kokoszynska (University of Lwow). I am especially indebted and grateful to Professors Ernest Nagel (Columbia University) and David Rynin (University of California, Berkeley) for their help in preparing the final text and for various critical remarks.

II. POLEMICAL REMARKS

14. IS THE SEMANTIC CONCEPTION OF TRUTH THE "RIGHT" ONE?

I should like to begin the polemical part of the paper with some general remarks.

I hope nothing which is said here will be interpreted as a claim that the semantic conception of truth is the "right" or indeed the "only possible" one. I do not have the slightest intention to contribute in any way to those endless, often violent discussions on the subject: "What is the right conception of truth?"²² I must confess I do not understand what is at stake in such disputes; for the problem itself is so vague that no definite solution is possible. In fact, it seems to me that the sense in which the phrase "the right conception" is used has never been made clear. In most cases one gets the impression that the phrase is used in an almost mystical sense based upon the belief that every word has only one "real" meaning (a kind of Platonic or Aristotelian idea), and that all the competing conceptions really attempt to catch hold of this one meaning; since, however, they contradict each other, only one attempt can be successful, and hence only one conception is the "right" one.

Disputes of this type are by no means restricted to the notion of truth. They occur in all domains where -- instead of an exact, scientific terminology -- common language with its vagueness and ambiguity is used; and they are always meaningless, and therefore in vain.

It seems to me obvious that the only rational approach to such problems would be the following: We should reconcile ourselves with the fact that we are confronted, not with one concept, but with several different concepts which are denoted by one word; we should try to make these concepts as clear as possible (by means of definition, or of an axiomatic procedure, or in some other way); to avoid further confusions, we should agree to use different terms for different concepts; and then we may proceed to a quiet and systematic study of all concepts involved, which will exhibit their main properties and mutual relations.

Referring specifically to the notion of truth, it is undoubtedly the case that in philosophical discussions -- and perhaps also in everyday usage -- some incipient conceptions of this notion can be found that differ essentially from the classical one (of which the semantic conception is but a modernized form). In fact, various conceptions

of this sort have been discussed in the literature, for instance, the pragmatic conception, the coherence theory, etc.⁴

It seems to me that none of these conceptions have been put so far in an intelligible and unequivocal form. This may change, however; a time may come when we find ourselves confronted with several incompatible, but equally clear and precise, conceptions of truth. It will then become necessary to abandon the ambiguous usage of the word "*true*," and to introduce several terms instead, each to denote a different notion. Personally, I should not feel hurt if a future world congress of the "theoreticians of truth" should decide -- by a majority of votes -- to reserve the word "*true*" for one of the non-classical conceptions, and should suggest another word, say, "*frue*," for the conception considered here. But I cannot imagine that anybody could present cogent arguments to the effect that the semantic conception is "wrong" and should be entirely abandoned.

15. FORMAL CORRECTNESS OF THE SUGGESTED DEFINITION OF TRUTH.

The specific objections which have been raised to my investigations can be divided into several groups; each of these will be discussed separately.

I think that practically all these objections apply, not to the special definition I have given, but to the semantic conception of truth in general. Even those which were leveled against the definition actually constructed could be related to any other definition which conforms to this conception.

This holds, in particular, for those objections which concern the formal correctness of the definition. I have heard a few objections of this kind; however, I doubt very much whether anyone of them can be treated seriously .

As a typical example let me quote in substance such an objection.²³ In formulating the definition we use necessarily sentential connectives, i.e., expressions like "*if* . . . , *then*," "*or*," etc. They occur in the definiens; and one of them, namely, the phrase "*if, and only if*" is usually employed to combine the definiendum with the definiens. However, it is well known that the meaning of sentential connectives is explained in logic with the help of the words "*true*" and "*false*"; for instance, we say that an equivalence, i.e., a sentence of the form "*p if, and only if, q*," is true if either both of its members, i.e., the sentences represented by '*p*' and '*q*,' are true or both are false. Hence the definition of truth involves a vicious circle.

If this objection were valid, no formally correct definition of truth would be possible; for we are unable to formulate any compound sentence without using sentential connectives, or other logical terms defined with their help. Fortunately, the situation is not so bad.

It is undoubtedly the case that a strictly deductive development of logic is often preceded by certain statements explaining the conditions under which sentences of the form "*if p, then q*," etc., are considered true or false. (Such explanations are often given

schematically, by means of the so-called truth-tables.) However, these statements are outside of the system of logic, and should not be regarded as definitions of the terms involved. They are not formulated in the language of the system, but constitute rather special consequences of the definition of truth given in the meta-language. Moreover, these statements do not influence the deductive development of logic in any way. For in such a development we do not discuss the question whether a given sentence is true, we are only interested in the problem whether it is provable.²⁴

On the other hand, the moment we find ourselves within the deductive system of logic -- or of any discipline based upon logic, e.g., of semantics -- we either treat sentential connectives as undefined terms, or else we define them by means of other sentential connectives, but never by means of semantic terms like "*true*" or "*false*." For instance, if we agree to regard the expressions "*not*" and "*if . . . , then*" (and possibly also "*if, and only if*") as undefined terms, we can define the term "*or*" by stating that a sentence of the form "*p or q*" is equivalent to the corresponding sentence of the form "*if not p, then q*." The definition can be formulated, e.g., in the following way:

(p or q) if, and only if, (if not p, then q).

This definition obviously contains no semantic terms.

However, a vicious circle in definition arises only when the definiens contains either the term to be defined itself, or other terms defined with its help. Thus we clearly see that the use of sentential connectives in defining the semantic term "*true*" does not involve any circle.

I should like to mention a further objection which I have found in the literature and which seems also to concern the formal correctness, if not of the definition of truth itself, then at least of the arguments which lead to this definition.²⁵

The author of this objection mistakenly regards scheme (T) (from [Section 4](#)) as a definition of truth. He charges this alleged definition with "inadmissible brevity, i.e., incompleteness," which "does not give us the means of deciding whether by 'equivalence' is meant a logical-formal, or a non-logical and also structurally non-describable relation." To remove this "defect" he suggests supplementing (T) in one of the two following ways:

(T') *X is true if, and only if, p is true,*

or

(T'') *X is true if, and only if, p is the case (i.e., if what p states is the case).*

Then he discusses these two new "definitions," which are supposedly free from the old, formal "defect," but which turn out to be unsatisfactory for other, non-formal reasons.

This new objection seems to arise from a misunderstanding concerning the nature of sentential connectives (and thus to be somehow related to that previously discussed). The author of the objection does not seem to realize that the phrase "*if, and only if*" (in opposition to such phrases as "*are equivalent*" or "*is equivalent to*") expresses no relation between sentences at all since it does not combine names of sentences.

In general, the whole argument is based upon an obvious confusion between sentences and their names. It suffices to point out that -- in contradistinction to (T) -- schemata (T') and (T'') do not give any meaningful expressions if we replace in them 'p' by a sentence; for the phrases "*p is true*" and "*p is the case*" (i.e., "*what p states is the case*") become meaningless if 'p' is replaced by a sentence, and not by the name of a sentence (cf. [Section 4](#)).²⁶

While the author of the objection considers schema (T) "inadmissible brief," I am inclined, on my part, to regard schemata (T') and (T'') as "inadmissibly long." And I think even that I can rigorously prove this statement on the basis of the following definition: An expression is said to be "inadmissibly long" if (i) it is meaningless, and (ii) it has been obtained from a meaningful expression by inserting superfluous words.

16. REDUNDANCY OF SEMANTIC TERMS -- THEIR POSSIBLE ELIMINATION.

The objection I am going to discuss now no longer concerns formal correctness of the definition, but is still concerned with certain formal features of the semantic conception of truth.

We have seen that this conception essentially consists in regarding the sentence "*X is true*" as equivalent to the sentence denoted by 'X' (where 'X' stands for a name of a sentence of the object language). Consequently, the term "*true*" when occurring in a simple sentence of the form "*X is true*" can easily be eliminated, and the sentence itself, which belongs to the meta-language, can be replaced by an equivalent sentence of the object-language; and the same applies to compound sentences provided the term "*true*" occurs in them exclusively as a part of the expressions of the form "*X is true*."

Some people have therefore urged that the term "*true*" in the semantic sense can always be eliminated, and that for this reason the semantic conception of truth is altogether sterile and useless. And since the same considerations apply to other semantic notions, the conclusion has been drawn that semantics as a whole is a purely verbal game and at best only a harmless hobby.

But the matter is not quite so simple.²⁷ The sort of elimination here discussed cannot always be made. It cannot be done in the case of universal statements which express the fact that all sentences of a certain type are true, or that all true sentences have a certain property. For instance, we can prove in the theory of truth the following statement:

All consequences of true sentences are true.

However, we cannot get rid here of the word "*true*" in the simple manner contemplated.

Again, even in the case of particular sentences having the form "*X is true*" such a simple elimination cannot always be made. In fact, the elimination is possible only in those cases in which the name of the sentence which is said to be true occurs in a form that enables us to reconstruct the sentence itself. For example, our present historical knowledge does not give us any possibility of eliminating the word "*true*" from the following sentence:

The first sentence written by Plato is true.

Of course, since we have a definition for truth and since every definition enables us to replace the definiendum by its definiens, an elimination of the term "*true*" in its semantic sense is always theoretically possible. But this would not be the kind of simple elimination discussed above, and it would not result in the replacement of a sentence in the meta-language by a sentence in the object-language.

If, however, anyone continues to urge that -- because of the theoretical possibility of eliminating the word "*true*" on the basis of its definition -- the concept of truth is sterile, he must accept the further conclusion that all defined notions are sterile. But this outcome is so absurd and so unsound historically that any comment on it is unnecessary. In fact, I am rather inclined to agree with those who maintain that the moments of greatest creative advancement in science frequently coincide with the introduction of new notions by means of definition.

17. CONFORMITY OF THE SEMANTIC CONCEPTION OF TRUTH WITH PHILOSOPHICAL AND COMMON- SENSE USAGE.

The question has been raised whether the semantic conception of truth can indeed be regarded as a precise form of the old, classical conception of this notion.

Various formulations of the classical conception were quoted in the early part of this paper ([Section 3](#)). I must repeat that in my judgment none of them is quite precise and clear. Accordingly, the only sure way of settling the question would be to confront the authors of those statements with our new formulation, and to ask them whether it agrees with their intentions. Unfortunately, this method is impractical since they died quite some time ago.

As far as my own opinion is concerned, I do not have any doubts that our formulation does conform to the intuitive content of that of Aristotle. I am less certain regarding the later formulations of the classical conception, for they are very vague indeed.²⁸

Furthermore, some doubts have been expressed whether the semantic conception does reflect the notion of truth in its common-sense and everyday usage. I clearly realize (as I already indicated) that the common meaning of the word "*true*" -- as that of any other word of everyday language -- is to some extent vague, and that its usage more or less fluctuates. Hence the problem of assigning to this word a fixed and exact meaning is relatively unspecified, and every solution of this problem implies necessarily a certain deviation from the practice of everyday language.

In spite of all this, I happen to believe that the semantic conception does conform to a very considerable extent with the common-sense usage -- although I readily admit I may be mistaken. What is more to the point, however, I believe that the issue raised can be settled scientifically, though of course not by a deductive procedure, but with the help of the statistical questionnaire method. As a matter of fact, such research has been

carried on, and some of the results have been reported at congresses and in part published.²⁹

I should like to emphasize that in my opinion such investigations must be conducted with the utmost care. Thus, if we ask a highschool boy, or even an adult intelligent man having no special philosophical training, whether he regards a sentence to be true if it agrees with reality, or if it designates an existing state of affairs, it may simply turn out that he does not understand the question; in consequence his response, whatever it may be, will be of no value for us. But his answer to the question whether he would admit that the sentence "*it is snowing*" could be true although it is not snowing, or could be false although it is snowing, would naturally be very significant for our problem.

Therefore, I was by no means surprised to learn (in a discussion devoted to these problems) that in a group of people who were questioned only 15% agreed that "*true*" means for them "*agreeing with reality*," while 90% agreed that a sentence such as "*it is snowing*" is true if, and only if, it is snowing. Thus, a great majority of these people seemed to reject the classical conception of truth in its "philosophical" formulation, while accepting the same conception when formulated in plain words (waiving the question whether the use of the phrase "the same conception" is here justified).

18. THE DEFINITION IN ITS RELATION TO "THE PHILOSOPHICAL PROBLEM OF TRUTH" AND TO VARIOUS EPISTEMOLOGICAL TRENDS.

I have heard it remarked that the formal definition of truth has nothing to do with "the philosophical problem of truth."³⁰ However, nobody has ever pointed out to me in an intelligible way just what this problem is. I have been informed in this connection that my definition, though it states necessary and sufficient conditions for a sentence to be true, does not really grasp the "essence" of this concept. Since I have never been able to understand what the "essence" of a concept is, I must be excused from discussing this point any longer.

In general, I do not believe that there is such a thing as "the philosophical problem of truth." I do believe that there are various intelligible and interesting (but not necessarily philosophical) problems concerning the notion of truth, but I also believe that they can be exactly formulated and possibly solved only on the basis of a precise conception of this notion.

While on the one hand the definition of truth has been blamed for not being philosophical enough, on the other a series of objections have been raised charging this definition with serious philosophical implications, always of a very undesirable nature. I shall discuss now one special objection of this type; another group of such objections will be dealt with in the next section.

It has been claimed that -- due to the fact that a sentence like "snow is white" is taken to be semantically true if snow is *in fact* white (italics by the critic) -- logic finds itself involved in a most uncritical realism.³¹

If there were an opportunity to discuss the objection with its author, I should raise two points. First, I should ask him to drop the words "*in fact*," which do not occur in the original formulation and which are misleading, even if they do not affect the content. For these words convey the impression that the semantic conception of truth is intended to establish the conditions under which we are warranted in asserting any given sentence, and in particular any empirical sentence. However, a moment's reflection shows that this impression is merely an illusion; and I think that the author of the objection falls victim to the illusion which he himself created.

In fact, the semantic definition of truth implies nothing regarding the conditions under which a sentence like (1):

(1) *snow is white*

can be asserted. It implies only that, whenever we assert or reject this sentence, we must be ready to assert or reject the correlated sentence (2):

(2) *the sentence "snow is white" is true*

Thus, we may accept the semantic conception of truth without giving up any epistemological attitude we may have had; we may remain naive realists, critical realists or idealists, empiricists or metaphysicians -- whatever we were before. The semantic conception is completely neutral toward all these issues.

In the second place, I should try to get some information regarding the conception of truth which (in the opinion of the author of the objection) does not involve logic in a most naive realism. I would gather that this conception must be incompatible with the semantic one. Thus, there must be sentences which are true in one of these conceptions without being true in the other. Assume, e.g., the sentence (1) to be of this kind. The truth of this sentence in the semantic conception is determined by an equivalence of the form (T):

The sentence "snow is white" is true if, and only if, snow is white.

Hence in the new conception we must reject this equivalence, and consequently we must assume its denial:

The sentence "snow is white" is true if, and only if, snow is not white (or perhaps: snow, in fact, is not white).

This sounds somewhat paradoxical. I do not regard such a consequence of the new conception as absurd; but I am a little fearful that someone in the future may charge this conception with involving logic in a "most sophisticated kind of irrationalism." At any rate, it seems to me important to realize that every conception of truth which is incompatible with the semantic one carries with it consequences of this type.

I have dwelt a little on this whole question, not because the objection discussed seems to me very significant, but because certain points which have arisen in the discussion should be taken into account by all those who for various epistemological reasons are inclined to reject the semantic conception of truth.

19. ALLEGED METAPHYSICAL ELEMENTS IN SEMANTICS.

The semantic conception of truth has been charged several times with involving certain metaphysical elements. Objections of this sort have been made to apply not only to the theory of truth, but to the whole domain of theoretical semantics.³²

I do not intend to discuss the general problem whether the introduction of a metaphysical element into a science is at all objectionable. The only point which will interest me here is whether and in what sense metaphysics is involved in the subject of our present discussion.

The whole question obviously depends upon what one understands by "metaphysics." Unfortunately, this notion is extremely vague and equivocal. When listening to discussions in this subject, sometimes one gets the impression that the term "metaphysical" has lost any objective meaning, and is merely used as a kind of professional philosophical invective.

For some people metaphysics is a general theory of objects (ontology) -- a discipline which is to be developed in a purely empirical way, and which differs from other empirical sciences only by its generality. I do not know whether such a discipline actually exists (some cynics claim that it is customary in philosophy to baptize unborn children); but I think that in any case metaphysics in this conception is not objectionable to anybody, and has hardly any connections with semantics.

For the most part, however, the term "metaphysical" is used as directly opposed - - in one sense or another to the term "empirical"; at any rate, it is used in this way by those people who are distressed by the thought that any metaphysical elements might have managed to creep into science. This general conception of metaphysics assumes several more specific forms.

Thus, some people take it to be symptomatic of a metaphysical element in a science when methods of inquiry are employed which are neither deductive nor empirical. However, no trace of this symptom can be found in the development of semantics (unless some metaphysical elements are involved in the object-language to which the semantic notions refer). In particular, the semantics of formalized languages is constructed in a purely deductive way.

Others maintain that the metaphysical character of a science depends mainly on its vocabulary and, more specifically, on its primitive terms. Thus, a term is said to be metaphysical if it is neither logical nor mathematical, and if it is not associated with an empirical procedure which enables us to decide whether a thing is denoted by this term or not. With respect to such a view of metaphysics it is sufficient to recall that a meta-language includes only three kinds of undefined terms: (i) terms taken from logic, (ii) terms of the corresponding object language, and (iii) names of expressions in the object-language. It is thus obvious that no metaphysical undefined terms occur in the meta-language (again, unless such terms appear in the object-language itself).

There are, however, some who believe that, even if no metaphysical terms occur among the primitive terms of a language, they may be introduced by definitions; namely, by those definitions which fail to provide us with general criteria for deciding whether an object falls under the defined concept. It is argued that the term "*true*" is of this kind, since no universal criterion of truth follows immediately from the definition

of this term, and since it is generally believed (and in a certain sense can even be proved) that such a criterion will never be found. This comment on the actual character of the notion of truth seems to be perfectly just. However, it should be noticed that the notion of truth does not differ in this respect from many notions in logic, mathematics, and theoretical parts of various empirical sciences, e.g., in theoretical physics.

In general, it must be said that if the term "metaphysical" is employed in so wide a sense as to embrace certain notions (or methods) of logic, mathematics, or empirical sciences, it will apply *a fortiori* to those of semantics. In fact, as we know from Part I of the paper, in developing the semantics of a language we use all the notions of this language, and we apply even a stronger logical apparatus than that which is used in the language itself. On the other hand, however, I can summarize the arguments given above by stating that in no interpretation of the term "metaphysical" which is familiar and more or less intelligible to me does semantics involve any metaphysical elements peculiar to itself.

I should like to make one final remark in connection with this group of objections. The history of science shows many instances of concepts which were judged metaphysical (in a loose, but in any case derogatory sense of this term) before their meaning was made precise; however, once they received a rigorous, formal definition, the distrust in them evaporated. As typical examples we may mention the concepts of negative and imaginary numbers in mathematics. I hope a similar fate awaits the concept of truth and other semantic concepts; and it seems to me, therefore, that those who have distrusted them because of their alleged metaphysical implications should welcome the fact that precise definitions of these concepts are now available. If in consequence semantic concepts lose philosophical interest, they will only share the fate of many other concepts of science, and this need give rise to no regret.

20. APPLICABILITY OF SEMANTICS TO SPECIAL EMPIRICAL SCIENCES.

We come to the last and perhaps the most important group of objections. Some strong doubts have been expressed whether semantic notions find or can find applications in various domains of intellectual activity. For the most part such doubts have concerned the applicability of semantics to the field of empirical science -- either to special sciences or to the general methodology of this field; although similar skepticism has been expressed regarding possible applications of semantics to mathematical sciences and their methodology.

I believe that it is possible to allay these doubts to a certain extent, and that some optimism with respect to the potential value of semantics for various domains of thought is not without ground.

To justify this optimism, it suffices I think to stress two rather obvious points. First, the development of a theory which formulates a precise definition of a notion and establishes its general properties provides *eo ipso* a firmer basis for all discussions in which this notion is involved; and, therefore, it cannot be irrelevant for anyone who uses this notion, and desires to do so in a conscious and consistent way. Secondly,

semantic notions are actually involved in various branches of science, and in particular of empirical science.

The fact that in empirical research we are concerned only with natural languages and that theoretical semantics applies to these languages only with certain approximation, does not affect the problem essentially. However, it has undoubtedly this effect that progress in semantics will have but a delayed and somewhat limited influence in this field. The situation with which we are confronted here does not differ essentially from that which arises when we apply laws of logic to arguments in everyday life -- or, generally, when we attempt to apply a theoretical science to empirical problems.

Semantic notions are undoubtedly involved, to a larger or smaller degree, in psychology, sociology, and in practically all the humanities. Thus, a psychologist defines the so-called intelligence quotient in terms of the numbers of *true* (right) and *false* (wrong) answers given by a person to certain questions; for a historian of culture the range of objects for which a human race in successive stages of its development possesses adequate *designations* may be a topic of great significance; a student of literature may be strongly interested in the problem whether a given author always uses two given words with the same *meaning*. Examples of this kind can be multiplied indefinitely.

The most natural and promising domain for the applications of theoretical semantics is clearly linguistics -- the empirical study of natural languages. Certain parts of this science are even referred to as "semantics," sometimes with an additional qualification. Thus, this name is occasionally given to that portion of grammar which attempts to classify all words of a language into parts of speech, according to what the words mean or designate. The study of the evolution of meanings in the historical development of a language is sometimes called "historical semantics." In general, the totality of investigations on semantic relations which occur in a natural language is referred to as "descriptive semantics." The relation between theoretical and descriptive semantics is analogous to that between pure and applied mathematics, or perhaps to that between theoretical and empirical physics; the role of formalized languages in semantics can be roughly compared to that of isolated systems in physics.

It is perhaps unnecessary to say that semantics cannot find any direct applications in natural sciences such as physics, biology, etc.; for in none of these sciences are we concerned with linguistic phenomena, and even less with semantic relations between linguistic expressions and objects to which these expressions refer. We shall see, however, in the next section that semantics may have a kind of indirect influence even on those sciences in which semantic notions are not directly involved.

21. APPLICABILITY OF SEMANTICS TO THE METHODOLOGY OF EMPIRICAL SCIENCE.

Besides linguistics, another important domain for possible applications of semantics is the methodology of science; this term is used here in a broad sense so as to embrace the theory of science in general. Independent of whether a science is conceived merely

as a system of statements or as a totality of certain statements and human activities, the study of scientific language constitutes an essential part of the methodological discussion of a science. And it seems to me clear that any tendency to eliminate semantic notions (like those of truth and designation) from this discussion would make it fragmentary and inadequate.³³ Moreover, there is no reason for such a tendency today, once the main difficulties in using semantic terms have been overcome. The semantics of scientific language should be simply included as a part in the methodology of science.

I am by no means inclined to charge methodology and, in particular, semantics -- whether theoretical or descriptive with the task of clarifying the meanings of all scientific terms. This task is left to those sciences in which the terms are used, and is actually fulfilled by them (in the same way in which, e.g., the task of clarifying the meaning of the term "true" is left to, and fulfilled by, semantics). There may be, however, certain special problems of this sort in which a methodological approach is desirable or indeed necessary (perhaps, the problem of the notion of causality is a good example here); and in a methodological discussion of such problems semantic notions may play an essential role. Thus, semantics may have some bearing on any science whatsoever.

The question arises whether semantics can be helpful in solving general and, so to speak, classical problems of methodology. I should like to discuss here with some detail a special, though very important, aspect of this question.

One of the main problems of the methodology of empirical science consists in establishing conditions under which an empirical theory or hypothesis should be regarded as acceptable. This notion of acceptability must be relativized to a given stage of the development of a science (or to a given amount of presupposed knowledge). In other words, we may consider it as provided with a time coefficient; for a theory which is acceptable today may become untenable tomorrow as a result of new scientific discoveries.

It seems *a priori* very plausible that the acceptability of a theory somehow depends on the truth of its sentences, and that consequently a methodologist in his (so far rather unsuccessful) attempts at making the notion of acceptability precise, can expect some help from the semantic theory of truth. Hence we ask the question: Are there any postulates which can be reasonably imposed on acceptable theories and which involve the notion of truth? And, in particular, we ask whether the following postulate is a reasonable one:

An acceptable theory cannot contain (or imply) any false sentences.

The answer to the last question is clearly negative. For, first of all, we are practically sure, on the basis of our historical experience, that every empirical theory which is accepted today will sooner or later be rejected and replaced by another theory. It is also very probable that the new theory will be incompatible with the old one; i.e., will imply a sentence which is contradictory to one of the sentences contained in the old theory. Hence, at least one of the two theories must include false sentences, in spite of the fact that each of them is accepted at a certain time. Secondly, the postulate in question could hardly ever be satisfied in practice; for we do not know, and are very unlikely to find,

any criteria of truth which enable us to show that no sentence of an empirical theory is false.

The postulate in question could be at most regarded as the expression of an ideal limit for successively more adequate theories in a given field of research; but this hardly can be given any precise meaning.

Nevertheless, it seems to me that there is an important postulate which can be reasonably imposed on acceptable empirical theories and which involves the notion of truth. It is closely related to the one just discussed, but is essentially weaker. Remembering that the notion of acceptability is provided with a time coefficient, we can give this postulate the following form:

As soon as we succeed in showing that an empirical theory contains (or implies) false sentences, it cannot be any longer considered acceptable.

In support of this postulate, I should like to make the following remarks.

I believe everybody agrees that one of the reasons which may compel us to reject an empirical theory is the proof of its inconsistency: a theory becomes untenable if we succeed in deriving from it two contradictory sentences. Now we can ask what are the usual motives for rejecting a theory on such grounds. Persons who are acquainted with modern logic are inclined to answer this question in the following way: A well-known logical law shows that a theory which enables us to derive two contradictory sentences enables us also to derive every sentence; therefore, such a theory is trivial and deprived of any scientific interest.

I have some doubts whether this answer contains an adequate analysis of the situation. I think that people who do not know modern logic are as little inclined to accept an inconsistent theory as those who are thoroughly familiar with it; and probably this applies even to those who regard (as some still do) the logical law on which the argument is based as a highly controversial issue, and almost as a paradox. I do not think that our attitude toward an inconsistent theory would change even if we decided for some reasons to weaken our system of logic so as to deprive ourselves of the possibility of deriving every sentence from any two contradictory sentences.

It seems to me that the real reason of our attitude is a different one: We know (if only intuitively) that an inconsistent theory must contain false sentences; and we are not inclined to regard as acceptable any theory which has been shown to contain such sentences.

There are various methods of showing that a given theory includes false sentences. Some of them are based upon purely logical properties of the theory involved; the method just discussed (i.e., the proof of inconsistency) is not the sole method of this type, but is the simplest one, and the one which is most frequently applied in practice. With the help of certain assumptions regarding the truth of empirical sentences, we can obtain methods to the same effect which are no longer of a purely logical nature. If we decide to accept the general postulate suggested above, then a successful application of any such method will make the theory untenable.

22. APPLICATIONS OF SEMANTICS TO DEDUCTIVE SCIENCE.

As regards the applicability of semantics to mathematical sciences and their methodology, i.e., to meta-mathematics, we are in a much more favorable position than in the case of empirical sciences. For, instead of advancing reasons which justify some hopes for the future (and thus making a kind of pro- semantics propaganda), we are able to point out concrete results already achieved.

Doubts continue to be expressed whether the notion of a true sentence -- as distinct from that of a provable sentence -- can have any significance for mathematical disciplines and play any part in a methodological discussion of mathematics. It seems to me, however, that just this notion of a true sentence constitutes a most valuable contribution to meta-mathematics by semantics. We already possess a sense of interesting meta-mathematical results gained with the help of the theory of truth. These results concern the mutual relations between the notion of truth and that of provability; establish new properties of the latter notion (which, as well known, is one of the basic notions of meta-mathematics); and throw some light on the fundamental problems of consistency and completeness. The most significant among these results have been briefly discussed in [Section 12](#).³⁴

Furthermore, by applying the method of semantics we can adequately define several important meta-mathematical notions which have been used so far only in an intuitive way -- such as, e.g., the notion of definability or that of a model of an axiom system; and thus we can undertake a systematic study of these notions. In particular, the investigations on definability have already brought some interesting results, and promise even more in the future.³⁵

We have discussed the applications of semantics only to meta-mathematics, and not to mathematics proper. However, this distinction between mathematics and meta-mathematics is rather unimportant. For meta-mathematics is itself a deductive discipline and hence, from a certain point of view, a part of mathematics; and it is well known that -- due to the formal character of deductive method -- the results obtained in one deductive discipline can be automatically extended to any other discipline in which the given one finds an interpretation. Thus, for example, all meta-mathematical results can be interpreted as results of number theory. Also from a practical point of view there is no clear-cut line between meta- mathematics and mathematics proper; for instance, the investigations on definability could be included in either of these domains.

23. FINAL REMARKS.

I should like to conclude this discussion with some general and rather loose remarks concerning the whole question of the evaluation of scientific achievements in terms of their applicability. I must confess I have various doubts in this connection.

Being a mathematician (as well as a logician, and perhaps a philosopher of a sort), I have had the opportunity to attend many discussions between specialists in

mathematics, where the problem of applications is especially acute, and I have noticed on several occasions the following phenomenon: If a mathematician wishes to disparage the work of one of his colleagues, say, A, the most effective method he finds for doing this is to ask where the results can be applied. The hard pressed man, with his back against the wall, finally unearths the researches of another mathematician B as the locus of the application of his own results. If next B is plagued with a similar question, he will refer to another mathematician C. After a few steps of this kind we find ourselves referred back to the researches of A, and in this way the chain closes.

Speaking more seriously, I do not wish to deny that the value of a man's work may be increased by its implications for the research of others and for practice. But I believe, nevertheless, that it is inimical to the progress of science to measure the importance of any research exclusively or chiefly in terms of its usefulness and applicability. We know from the history of science that many important results and discoveries have had to wait centuries before they were applied in any field. And, in my opinion, there are also other important factors which cannot be disregarded in determining the value of a scientific work. It seems to me that there is a special domain of very profound and strong human needs related to scientific research, which are similar in many ways to aesthetic and perhaps religious needs. And it also seems to me that the satisfaction of these needs should be considered an important task of research. Hence, I believe, the question of the value of any research cannot be adequately answered without taking into account the intellectual satisfaction which the results of that research bring to those who understand it and care for it. It may be unpopular and out-of-date to say -- but I do not think that a scientific result which gives a better understanding of the world and makes it more harmonious in our eyes should be held in lower esteem than, say, an invention which reduces the cost of paving roads, or improves household plumbing.

It is clear that the remarks just made become pointless if the word "application" is used in a very wide and liberal sense. It is perhaps not less obvious that nothing follows from these general remarks concerning the specific topics which have been discussed in this paper; and I really do not know whether research in semantics stands to gain or lose by introducing the standard of value I have suggested.

NOTES

¹ Compare [Tarski \[2\]](#) (see bibliography at the end of the paper). This work may be consulted for a more detailed and formal presentation of the subject of the paper, especially of the material included in [Sections 6 and 9-13](#). It contains also references to my earlier publications on the problems of semantics (a communication in Polish 1930; the article [Tarski \[1\]](#) in French, 1931; a communication in German, 1932; and a book in Polish, 1933). The expository part of the present paper is related in its character to [Tarski \[3\]](#). My investigations on the notion of truth and on theoretical semantics have been reviewed or discussed in [Hofstadter \[1\]](#), [Juhos \[1\]](#), [Kokoszynska \[1\]](#) and [\[2\]](#), [Kotarbinski \[2\]](#), [Scholz \[1\]](#), [Weinberg \[1\]](#), *et al.*

² It may be hoped that the interest in theoretical semantics will now increase, as a result of the recent publication of the important work [Carnap \[2\]](#).

³ This applies, in particular, to public discussions during the I. International Congress for the Unity of Science (Paris, 1935) and the Conference of International Congresses for the Unity of Science (Paris, 1937); cf., e.g., [Neurath \[1\]](#) and [Gonseth \[1\]](#).

⁴ The words "notion" and "concept" are used in this paper with all of the vagueness and ambiguity with which they occur in philosophical literature. Thus, sometimes they refer simply to a term, sometimes to what is meant by a term, and in other cases to what is denoted by a term. Sometimes it is irrelevant which of these interpretations is meant; and in certain cases perhaps none of them applies adequately. While on principle I share the tendency to avoid these words in any exact discussion, I did not consider it necessary to do so in this informal presentation.

⁵ For our present purposes it is somewhat more convenient to understand by "expressions," "sentences," etc., not individual inscriptions, but classes of inscriptions of similar form (thus, not individual physical things, but classes of such things).

⁶ For the Aristotelian formulation see [Aristotle \[1\]](#), gamma, 7, 27. The other two formulations are very common in the literature, but I do not know with whom they originate. A critical discussion of various conceptions of truth can be found, e.g., in [Kotarbinski \[1\]](#) (so far available only in Polish), pp. 123 ff., and [Russell \[1\]](#), pp. 362 ff.

⁷ For most of the remarks contained in [Sections 4](#) and [8](#), I am indebted to the late S. Lesniewski who developed them in his unpublished lectures in the University of Warsaw (in 1919 and later). However, Lesniewski did not anticipate the possibility of a rigorous development of the theory of truth, and still less of a definition of this notion; hence, while indicating equivalences of the form (T) as premises in the antinomy of the liar, he did not conceive them as any sufficient conditions for an adequate usage (or definition) of the notion of truth. Also the remarks in [Section 8](#) regarding the occurrence of an empirical premise in the antinomy of the liar, and the possibility of eliminating this premiss, do not originate with him.

⁸ In connection with various logical and methodological problems involved in this paper the reader may consult [Tarski \[6\]](#).

⁹ The antinomy of the liar (ascribed to Eubulides or Epimenides) is discussed here in [Sections 7](#) and [8](#). For the antinomy of definability (due to J. Richard) see, e.g., [Hilbert-Bernays \[1\]](#), vol. 2, pp. 263 ff.; for the antinomy of heterological terms see [Grelling-Nelson \[1\]](#), p. 307.

¹⁰ Due to Professor J. Lukasiewicz (University of Warsaw).

¹¹ This can roughly be done in the following way. Let S be any sentence beginning with the words "Every sentence." We correlate with S a new sentence S^* by subjecting S to the following two modifications: we replace in S the first word, "Every," by "The"; and we insert after the second word, "sentence," the whole sentence S enclosed in quotation marks. Let us agree to call the sentence S "(self-)applicable" or "non-(self-)applicable" dependent on whether the correlated sentence S^* is true or false. Now consider the following sentence:

Every sentence is non-applicable.

It can easily be shown that the sentence just stated must be both applicable and non-applicable; hence a contradiction. It may not be quite clear in what sense this formulation of the antinomy does not involve an empirical premiss; however, I shall not elaborate on this point.

¹² The terms "logic" and "logical" are used in this paper in a broad sense, which has become almost traditional in the last decades; logic is assumed here to comprehend the whole theory of classes and relations (i.e., the mathematical theory of sets). For many different reasons I am personally inclined to use the term "logic" in a much narrower sense, so as to apply it only to what is sometimes called "elementary logic," i.e., to the sentential calculus and the (restricted) predicate calculus.

¹³ Cf. here, however, [Tarski \[3\]](#), p. 5 f.

¹⁴ The method of construction we are going to outline can be applied -- with appropriate changes -- to all formalized languages that are known at the present time; although it does not follow that a language could not be constructed to which this method would not apply.

¹⁵ In carrying through this idea a certain technical difficulty arises. A sentential function may contain an arbitrary number of free variables; and the logical nature of the notion of satisfaction varies with this number. Thus, the notion in question when applied to functions with one variable is a binary relation between these functions and single objects; when applied to functions with two variables it becomes a ternary relation between functions and couples of objects; and soon. Hence, strictly speaking, we are confronted, not with one notion of satisfaction, but with infinitely many notions; and it turns out that these notions cannot be defined independently of each other, but must all be introduced simultaneously.

To overcome this difficulty, we employ the mathematical notion of an infinite sequence (or, possibly, of a finite sequence with an arbitrary number of terms). We agree to regard satisfaction, not as a many-termed relation between sentential functions and an indefinite number of objects, but as a binary relation between functions and sequences of objects. Under this assumption the formulation of a general and precise definition of satisfaction no longer presents any difficulty; and a true sentence can now be defined as one which is satisfied by every sequence.

¹⁶ To define recursively the notion of satisfaction, we have to apply a certain form of recursive definition which is not admitted in the object-language. Hence the "essential richness" of the meta-language may simply consist in admitting this type of definition. On the other hand, a general method is known which makes it possible to eliminate all recursive definitions and to replace them by normal, explicit ones. If we try to apply this method to the definition of satisfaction, we see that we have either to introduce into the meta-language variables of a higher logical type than those which occur in the object-language; or else to assume axiomatically in the meta-language the existence of classes that are more comprehensive than all those whose existence can be established in the object-language. See here [Tarski \[2\]](#), pp. 393 ff., and [Tarski \[5\]](#), p. 110.

¹⁷ Due to the development of modern logic, the notion of mathematical proof has undergone far-reaching simplification. A sentence of a given formalized discipline is

provable if it can be obtained from the axioms of this discipline by applying certain simple and purely formal rules of inference, such as those of detachment and substitution. Hence to show that all provable sentences are true, it suffices to prove that all the sentences accepted as axioms are true, and that the rules of inference when applied to true sentences yield new true sentences; and this usually presents no difficulty.

On the other hand, in view of the elementary nature of the notion of provability, a precise definition of this notion requires only rather simple logical devices. In most cases, those logical devices which are available in the formalized discipline itself (to which the notion of provability is related) are more than sufficient for this purpose. We know, however, that as regards the definition of truth just the opposite holds. Hence, as a rule, the notions of truth and provability cannot coincide; and since every provable sentence is true, there must be true sentences which are not provable.

¹⁸ Thus the theory of truth provides us with a general method for consistency proofs for formalized mathematical disciplines. It can be easily realized, however, that a consistency proof obtained by this method may possess some intuitive value -- i.e., may convince us, or strengthen our belief, that the discipline under consideration is actually consistent -- only in case we succeed in defining truth in terms of a meta-language which does not contain the object-language as a part (cf. here a remark in [Section 9](#)). For only in this case the deductive assumptions of the meta-language may be intuitively simpler and more obvious than those of the object-language -- even though the condition of "essential richness" will be formally satisfied. Cf. here also [Tarski \[3\]](#), p. 7.

The incompleteness of a comprehensive class of formalized disciplines constitutes the essential content of a fundamental theorem of K. [Gödel \[1\]](#), pp. 187 ff. The explanation of the fact that the theory of truth leads so directly to Gödel's theorem is rather simple. In deriving Gödel's result from the theory of truth we make an essential use of the fact that the definition of truth cannot be given in a meta-language which is only as "rich" as the object-language (cf. [note 17](#)); however, in establishing this fact, a method of reasoning has been applied which is very closely related to that used (for the first time) by Gödel. It may be added that Gödel was clearly guided in his proof by certain intuitive considerations regarding the notion of truth, although this notion does not occur in the proof explicitly; cf. [Gödel \[1\]](#), pp. 174 f.

¹⁹ The notions of designation and definition lead respectively to the antinomies of Grelling-Nelson and Richard (cf. [note 9](#)). To obtain an antinomy for the notion of satisfaction, we construct the following expression:

The sentential function X does not satisfy X.

A contradiction arises when we consider the question whether this expression, which is clearly a sentential function, satisfies itself or not.

²⁰ All notions mentioned in this section can be defined in terms of satisfaction. We can say, e.g., that a given term designates a given object if this object satisfies the sentential function "*x is identical with T*" where '*T*' stands for the given term. Similarly, a sentential function is said to define a given object if the latter is the only object which

satisfies this function. For a definition of consequence see [Tarski \[4\]](#), and for that of synonymy -- [Carnap \[2\]](#).

²¹ General semantics is the subject of [Carnap \[2\]](#). Cf. here also remarks in [Tarski \[2\]](#), pp. 388 f.

²² Cf. various quotations in [Ness \[1\]](#), pp. 13 f.

²³ The names of persons who have raised objections will not be quoted here, unless their objections have appeared in print.

²⁴ It should be emphasized, however, that as regards the question of an alleged vicious circle the situation would not change even if we took a different point of view, represented, e.g., in [Carnap \[2\]](#); i.e., if we regarded the specification of conditions under which sentences of a language are true as an essential part of the description of this language. On the other hand, it may be noticed that the point of view represented in the text does not exclude the possibility of using truth-tables in a deductive development of logic. However, these tables are to be regarded then merely as a formal instrument for checking the provability of certain sentences; and the symbols '*T*' and '*F*' which occur in them and which are usually considered abbreviations of "*true*" and "*false*" should not be interpreted in any intuitive way.

²⁵ Cf. [Juhos \[1\]](#). I must admit that I do not clearly understand von Juhos' objections and do not know how to classify them; therefore, I confine myself here to certain points of a formal character. Von Juhos does not seem to know my definition of truth; he refers only to an informal presentation in [Tarski \[3\]](#) where the definition has not been given at all. If he knew the actual definition, he would have to change his argument. However, I have no doubt that he would discover in this definition some "defects" as well. For he believes he has proved that "on ground of principle it is impossible to give such a definition at all."

²⁶ The phrases "*p is true*" and "*p is the case*" (or better "*it is true that p*" and "*it is the case that p*") are sometimes used in informal discussions, mainly for stylistic reasons; but they are considered then as synonymous with the sentence represented by '*p*'. On the other hand, as far as I understand the situation, the phrases in question cannot be used by von Juhos synonymously with '*p*'; for otherwise the replacement of (T) by (T') or (T'') would not constitute any "improvement."

²⁷ Cf. the discussion of this problem in [Kokoszynska \[1\]](#), pp. 161 ff.

²⁸ Most authors who have discussed my work on the notion of truth are of the opinion that my definition does conform with the classical conception of this notion; see, e.g., [Kotarbinski \[2\]](#) and [Scholz \[1\]](#).

²⁹ Cf. [Ness \[1\]](#). Unfortunately, the results of that part of Ness' research which is especially relevant for our problem are not discussed in his book; compare p. 148, footnote 1.

³⁰ Though I have heard this opinion several times, I have seen it in print only once and, curiously enough, in a work which does not have a philosophical character -- in fact, in

[Hilbert-Bernays \[1\]](#), vol. II, p. 269 (where, by the way, it is not expressed as any kind of objection). On the other hand, I have not found any remark to this effect in discussions of my work by professional philosophers (cf. [note 1](#)).

³¹ Cf. [Gonseth \[1\]](#), pp. 187 f.

³² See [Nagel \[1\]](#), and [Nagel \[2\]](#), pp. 471 f. A remark which goes, perhaps, in the same direction is also to be found in [Weinberg \[1\]](#), p. 77; cf., however, his earlier remarks, pp. 75 f.

³³ Such a tendency was evident in earlier works of Carnap (see, e.g., [Carnap \[1\]](#), especially Part V) and in writings of other members of Vienna Circle. Cf. here [Kokoszynska \[1\]](#) and [Weinberg \[1\]](#).

³⁴ For other results obtained with the help of the theory of truth see [Gödel \[2\]](#); [Tarski \[2\]](#), pp. 401 ff.; and [Tarski \[5\]](#), pp. 111 f.

³⁵ An object -- e.g., a number or a set of numbers -- is said to be definable (in a given formalism) if there is a sentential function which defines it; cf. [note 20](#). Thus, the term "definable," though of a meta-mathematical (semantic) origin, is purely mathematical as to its extension, for it expresses a property (denotes a class) of mathematical objects. In consequence, the notion of definability can be re-defined in purely mathematical terms, though not within the formalized discipline to which this notion refers; however, the fundamental idea of the definition remains unchanged. Cf. here -- also for further bibliographic reference -- [Tarski \[1\]](#); various other results concerning definability can also be found in the literature, e.g., in [Hilbert-Bernays \[1\]](#), vol. I, pp. 354 ff., 369 ff., 456 ff., etc., and in [Lindenbaum-Tarski \[1\]](#). It may be noticed that the term "definable" is sometimes used in another, meta-mathematical (but not semantic), sense; this occurs, for instance, when we say that a term is definable in other terms (on the basis of a given axiom system). For a definition of a model of an axiom system see [Tarski \[4\]](#).

BIBLIOGRAPHY

Only the books and articles actually referred to in the paper will be listed here.

Aristotle. [1].

Metaphysica. (Works, vol. VIII.) English translation by W. D. Ross. Oxford, 1908.

Carnap, R. [1].

Logical Syntax of Language. London and New York, 1937.

Carnap, R. [2].

Introduction to Semantics. Cambridge, 1942.

Gödel, K. [1].

"Über formal unentscheidbare Sätze der *Principia Mathematica* und verwandter Systeme, I." *Monatshefte für Mathematik und Physik*, vol. XXXVIII, 1931, pp. 173-198.

Gödel, K. [2].

"Über die Länge von Beweisen." *Ergebnisse eines mathematischen Kolloquiums*, vol. VII, 1936, pp. 23-24.

Gonseth, F. [1].

- "Le Congrès Descartes. Questions de Philosophie scientifique." *Revue thomiste*, vol. XLIV, 1938, pp. 183-193.
- Grelling, K., and Nelson, L. [1].
"Bemerkungen zu den Paradoxien von Russell und Burali-Forti." *Abhandlungen der Fries'schen Schule*, vol. II (new series), 1908, pp. 301-334.
- Hofstadter, A. [1].
"On Semantic Problems." *The Journal of Philosophy*, vol. XXXV, 1938, pp. 225-232.
- Hilbert, D., and Bernays, P. [1].
Grundlagen der Mathematik. 2 vols. Berlin, 1934-1939.
- Juhos, B. von.[1].
"The Truth of Empirical Statements." *Analysis*, vol. IV, 1937, pp. 65-70.
- Kokoszynska, M. [1].
"Über den absoluten Wahrheitsbegriff und einige andere semantische Begriffe." *Erkenntnis*, vol. VI, 1936, pp. 143-165.
- Kokoszynska, M. [2].
"Syntax, Semantik und Wissenschaftslogik." *Actes du Congrès International de Philosophie Scientifique*, vol. III, Paris, 1936, pp. 9-14.
- Kotarbinski, T. [1].
Elementy teorii poznania, logiki formalnej i metodologii nauk. (Elements of Epistemology, Formal Logic, and the Methodology of Sciences, in Polish.) Lwow, 1929.
- Kotarbinski, T. [2].
"W sprawie pojecia prawdy." ("Concerning the Concept of Truth," in Polish.) *Przegląd filozoficzny*, vol. XXXVII, pp. 85-91.
- Lindenbaum, A., and Tarski, A. [1].
"Über die Beschränktheit der Ausdrucksmittel deduktiver Theorien." *Ergebnisse eines mathematischen Kolloquiums*, vol. VII, 1936, pp. 15-23.
- Nagel, E. [1].
Review of [Hofstadter \[1\]](#). *The Journal of Symbolic Logic*, vol. III, 1938, p. 90.
- Nagel, E. [2].
Review of [Carnap \[2\]](#). *The Journal of Philosophy*, vol. XXXIX, 1942, pp. 468-473.
- Ness, A. [1].
" 'Truth' As Conceived by Those Who Are Not Professional Philosophers." *Skifter utgitt av Det Norske Videnskaps-Akademi i Oslo, II. Hist. Filos. Klasse*, vol. IV, Oslo, 1938.
- Neurath, O. [1].
"Erster Internationaler Kongress für Einheit der Wissenschaft in Paris 1936." *Erkenntnis*, vol. V, 1935, pp. 377-406.
- Russell, B. [1].
An Inquiry Into Meaning and Truth. New York, 1940.
- Scholz, H. [1].
Review of *Studia philosophica*, vol. I. *Deutsche Literaturzeitung*, vol. LVIII, 1937, pp. 1914-1917.
- Tarski, A. [1].
"Sur les ensembles définissables de nombres réels. I." *Fundamenta mathematicae*, vol, XVII, 1911, pp, 210-239.
- Tarski, A. [2].

- "Der Wahrheitsbegriff in den formalisierten Sprachen." (German translation of a book in Polish, 1933.) *Studia philosophica*, vol. I, 1935, pp. 261-405.
- Tarski, A. [3].
"Grundlegung der wissenschaftlichen Semantik." *Actes du Congrès International de Philosophie Scientifique*, vol. III, Paris, 1936, pp. 1-8.
- Tarski, A. [4].
"Über den Begriff der logischen Folgerung." *Actes du Congrès International de Philosophie Scientifique*, vol. VII, Paris, 1937, pp. 1-11.
- Tarski, A. [5].
"On Undecidable Statements in Enlarged Systems of Logic and the Concept of Truth." *The Journal of Symbolic Logic*, vol. IV, 1939, pp. 105-112.
- Tarski, A. [6].
Introduction to Logic. New York, 1941.
- Weinberg, J. [1].
Review of *Studia Philosophica*, vol. I. *The Philosophical Review*, vol. XLVII, pp. 70-77.
-